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MHD nanofluid flow over an exponentially stretching surface with suction/injection

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Abstract-Present study deals with the heat and mass transfer in MHD nanofluid flow over an exponentially stretching surface in presence of internal heat generation/absorption, chemical reaction, radiation and suction/injection effects. The governing partial differential equations of the flow are converted into nonlinear coupled ordinary differential equations by using similarity transformation. Shooting technique is employed to yield the numerical solutions for the model. The effect of non-dimensional parameters on velocity, temperature and concentration profiles are discussed and presented through graphs.

Index Terms: MHD, Suction/injection, Heat generation/absorption, Nanofluid, Convection, Radiation.

1 INTRODUCTION

The process of heat and mass transfer is encountered in aeronautics, fluid fuel nuclear reactor, chemical process industries and many engineering applications in which the fluid is the working medium. Nanofluid is a fluid containing nanometer-sized particles, called nanoparticles. These fluids are engineered colloidal suspensions of nanoparticles in a base fluid. Chakrabarti and Gupta [1] analyzed MHD flow and heat transfer characteristics over a stretching surface. Ahmad et al. [2] discussed MHD flow of a nanofluid over an exponentially stretching sheet. Rohni et al. [3] studied stagnation pot flow over an exponentially shrinking sheet with suction effect.

Radiation and chemical reaction effects of MHD flow over a vertical plate in porous medium was studied by Sandeep et al. [4]. Influence of aligned magneticfiled on unsteady free convective flow past parallel flat plates was discussed by Sandeep and Sugunamma [5]. Makinde and Aziz [6] analyzed boundary layer flow of a nanofluid past a stretching sheet with convective boundary conditions. Mohan Krishna et al. [7] illustrated radiation effect on unsteady natural convective flow of a nanofluid past a parallel plate. Researchers [8-11] discussed radiation and chemical reaction effects on the flow through different channels by considering nano and dusty fluids. Sandeep et al. [12] analyzed magneticfield, radiation and rotation effect on the flow over moving vertical plate in porous medium. Rana and Bhargava [13] discussed flow and heat transfer characteristics of a nanofluid over a nonlinear stretching
surface.

Bhattacharya [14] illustrated boundary layer flow and heat transfer over exponentially shrinking surface. Ferdows et al. [15] analyzed boundary layer flow and heat transfer over a permeable unsteady stretching sheet. Very recently the researchers [16-19] presented influence of radiation and chemical reaction on some base and nanofluids at different channels. Subhas Abel et al. [20] presented a numerical solution for the momentum and heat transfer equations of hydromantic flow due to a stretching sheet.

In this study we analyzed the heat and mass transfer in MHD nanofluid flow over an exponentially stretching surface in presence of internal heat generation/absorption, chemical reaction, radiation and suction/injection effects. The governing partial differential equations of the flow are converted into nonlinear coupled ordinary differential equations by using similarity transformation and then solved numerically. The effect of non-dimensional parameters on velocity, temperature and concentration profiles are discussed and presented through graphs.

2 MATHEMATICAL FORMULATION

Consider a steady, incompressible, electrically conducting, two dimensional boundary layer flow of a dissipative nanofluid an over exponentially stretching sheet in a porous medium. The x-axis is along the continuous stretching surface and y axis is normal to the surface. The porous medium with non uniform permeability k is considered and a variable magneticfield $B(x)$ is applied along y direction. A variable heat source $Q(x)$, Suction/injection effects along with thermophoretic is taken in to account. The boundary layer equations that governs the present flow subject to the Boussinesq approximations can be expressed as

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$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho_{nf}}\begin{bmatrix}\mu_{nf}\frac{\partial^2 u}{\partial y^2}+g(\rho\beta)_{nf}(T-T_\infty)\\ -\sigma B^2(x)u-\frac{\upsilon_f}{k}u\end{bmatrix} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}-\frac{1}{(\rho c_p)_{nf}}\frac{\partial q_r}{\partial y}+\frac{Q(x)}{(\rho c_p)_{nf}}(T-T_\infty) \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}-K_l(C-C_\infty) \tag{4}$$

Where u and v are the velocity components in the x, y directions , ρ_{nf} is the density of nanofluid, μ_{nf} is the dynamic viscosity of the nanofluid, g is the acceleration due to gravity , β_{nf} is the volumetric coefficient of thermal expansion, σ is the electrical conductivity, $B(x)=B_0\,e^{Nx/2L}$ is the variable magnetic field, B_0 is the constant magneticfiled, N is the exponential parameter, υ_f is the kinematic viscosity of the fluid, $k=k_0e^{-Nx/L}$ is the non uniform permeability of porous medium, T and T_∞ are the surface and ambient temperatures, $\alpha_{nf}=k_{nf}/(\rho c_p)_{nf}$ is the thermal diffusivity, $(c_p)_{nf}$ is the specific heat of the nanofluid, q_r is the radiative heat flux, $Q(x)=Q_0e^{Nx/L}$ is the internal heat source/sink, D_m is the molecular diffusivity of species concentration,

The boundary conditions of the flow is given by

$$\left.\begin{array}{ll} u=u_w(x), v=\pm v_w(x), \mathrm{T}=\mathrm{T}_w, C=C_w & at\ \ y=0\\ u\to 0, T\to T_\infty, C\to C_\infty & as\ \ y\to\infty\end{array}\right\} \tag{5}$$

Where $u=u_w(x)=U_0e^{Nx/L}$ is the surface velocity and $v_w(x)=v_0e^{Nx/2L}$ is the special type of velocity at the surface. Here $v_w(x)>0$ represents suction and $v_w(x)<0$ represents injection on the porous surface.

The radiative heat flux q_r under Rosseland approximation is of the form

$$q_r=-\frac{4\sigma_1}{3\chi}\frac{\partial T^4}{\partial y} \tag{6}$$

where σ_1 is the Stefan-Boltzmann constant and χ is the mean absorption coefficient. The temperature differences within the flow are assumed to be sufficiently small such that T^4 may be expressed as a linear function of temperature. Expanding T^4 using Taylor series and neglecting higher order terms yields

$$T^4\cong 4T_\infty{}^3T-3T_\infty{}^4 \tag{7}$$

Substituting equations (6) and (7) into (3), we get

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma_1T_\infty^3}{3(\rho c_p)_{nf}\,\chi}\frac{\partial^2 T}{\partial y^2}+\frac{Q(x)}{(\rho c_p)_{nf}}(T-T_\infty) \tag{8}$$

The governing equations (1)-(4) and (8) can be simplified by introducing the similarity transformations

$$\left.\begin{array}{l}\eta=y\sqrt{U_0/2\upsilon_{nf}L}e^{Nx/2L},\ u=U_0e^{Nx/L}f'(\eta),\\ C=C_0e^{Nx/2L}\phi(\eta),\\ T=T_w=T_\infty+T_0e^{Nx/2L}\theta(\eta),\\ v=-N\sqrt{\upsilon_{nf}U_0/2L}\,e^{Nx/2L}\{f(\eta)+\eta f'(\eta)\}\end{array}\right. \tag{9}$$

Using (9) the governing partial differential equations are reduced to

$$f'''+Nff''-2Nf'^2+Gr\theta-(M+K)f'=0 \tag{10}$$

$$\left(\frac{1}{\Pr}+R\right)\theta''+N(f\theta'-\theta f')+Ec(f'')^2+Q_H\theta=0 \tag{11}$$

$$\phi''+NSc(f\phi'-f'\phi)-ScK_l\phi=0 \tag{12}$$

Subject to the boundary conditions

$$\begin{array}{l} f=f_w, f'=1, \theta=1, \phi=1\ \ at\ \eta=0\\ f'\to 0, \theta\to 0, \phi\to 0,\ \ as\ \eta\to\infty\end{array} \tag{13}$$

Where N is the exponential parameter, $Gr=2Lg\beta_{nf}T_0/U_0^2$ is the thermal Grashof number, $M=2L\sigma B_0{}^2/\rho_{nf}U_0$ is the Hartmann number , $K=2L\upsilon_{nf}/k_0U_0$ is the porosity parameter, $\Pr=\upsilon_{nf}/\alpha_{nf}$ is the Prandtl number , $R=16\sigma_1T_\infty{}^3/3\chi(\mu c_p)_{nf}$ is the Radiation parameter, $Ec=U_0{}^2/T_0(c_p)_{nf}$ is the Eckert number, $Q_H=2LQ_0/(\rho c_p)_{nf}U_0$ is the internal heat source /sink, $Sc=\upsilon_{nf}/D_m$ is the Schmidt number, $K_l=k_lC_0/U_0L$ is the chemical reaction parameter and $f_w=-v_w(x)\sqrt{\upsilon_{nf}U_0/2\mathrm{L}}$ is the suction/injection parameter with positive value indicates suction while negative value indicates injection.

3 RESULTS AND DISCUSSION

Equations (10) to (12) with the boundary conditions (13) have been solved numerically. The results obtained shows the influences of the non dimensional governing parameters, namely Magneticfield parameter M, Heat generation/absorption parameter Q_H, Exponential parameter N, Radiation parameter R, Eckert number Ec, Chemical reaction parameter K_l and Suction/injection parameter f_w, on the velocity, temperature and concentration profiles. In this study for numerical results we considered the non dimensional parameter values as $\Pr = 6.2, N = 2, \mathrm{Gr} = 5$, $\mathrm{M} = \mathrm{R} = Q_H = 1, Sc = 0.2, Ec = 0.1, K = K_l = f_w = 0.5$. These values are kept as constant except the varied parameters as shown in figures.

Figs. 1 and 2 illustrate the effect exponential parameter (N) on velocity, temperature and concentration profiles. It is evident from figures that an increase in exponential parameter depreciates the velocity, temperature and concentration profiles. Physically this means that increase in N value reduces the momentum, thermal and concentration boundary layer thickness and it is important to mention here that for positive exponential parameter also the surface temperature depreciates near the boundary layer.

Figs. 3 and 4 show the effect magneticfield parameter (M) on the velocity, temperature and concentration profiles. It is noticed from figures that increase in magneticfiled parameter decreases the velocity profiles but it is reversed in temperature and concentration profiles. It is due to the fact that increase in magneticfield generates the opposite force to the flow, is called Lorentz force. This force helps to enhance the thermal and concentration boundary layers. Due to this reason we have seen rising in the values of temperature and concentration profiles.

Figs. 5 and 6 represent the effect of radiation parameter (R) on the velocity, temperature and concentration profiles. It is clear from figures that nanofluid velocity and temperature distributions increase with increases in radiation parameter but it shows opposite action in concentration profiles. This agrees the physical behavior that at $R \neq 0$ the radiation is more significant and it causes to momentum and thermal boundary layers become thinner. Figs. 7 and 8 illustrate the effect of heat generation/absorption parameter on the nanofluid velocity, temperature and concentration profiles. The results obtained in this case are similar to the results observed for radiation parameter.

Figs. 9 and 10 depict the effect viscous dissipation parameter and chemical reaction parameter on temperature and concentration profiles. It is noticed from figures that increase in dissipation parameter increases the temperature profiles and depreciates the concentration profiles. And enhances in chemical reaction parameter declines the concentration profiles. But it does not shown any influence in temperature profiles. Figs. 11 and 12 display the influence of suction/injection parameter on velocity, temperature and concentration profiles. It is clear from the figures that increase in suction/injection parameter reduces the velocity, temperature and concentration boundary layer.

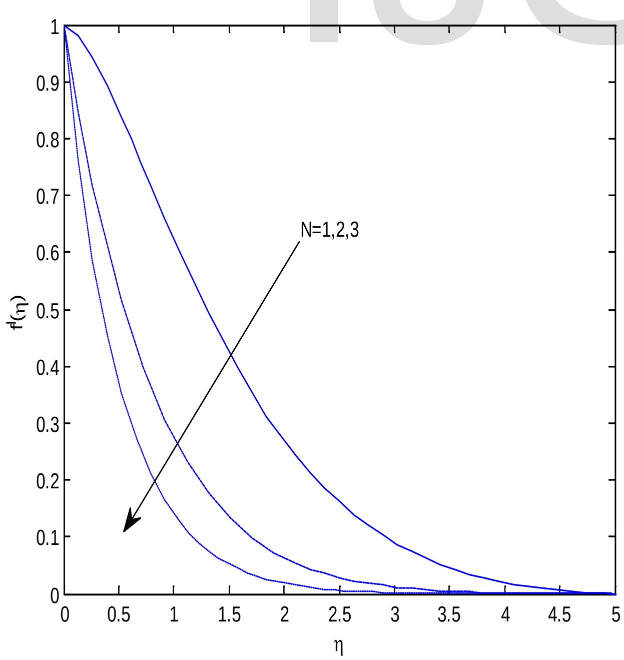

Fig. 1 Velocity profiles for different values of exponential parameter N

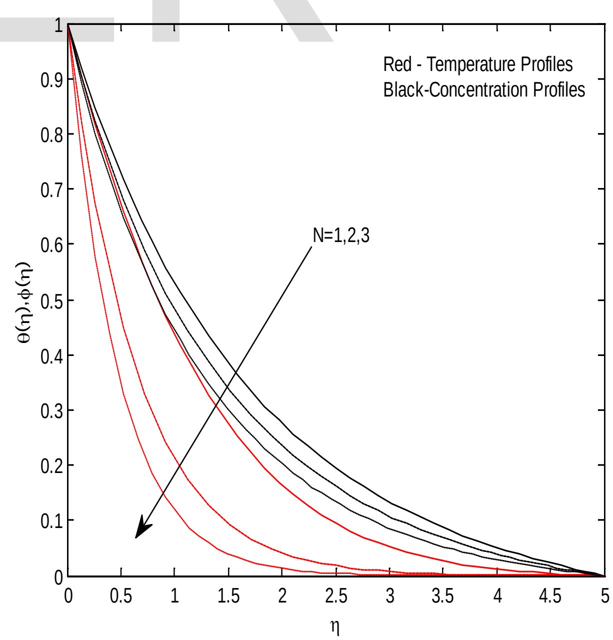

Fig. 2 Temperature and Concentration profiles for different values of exponential parameter N

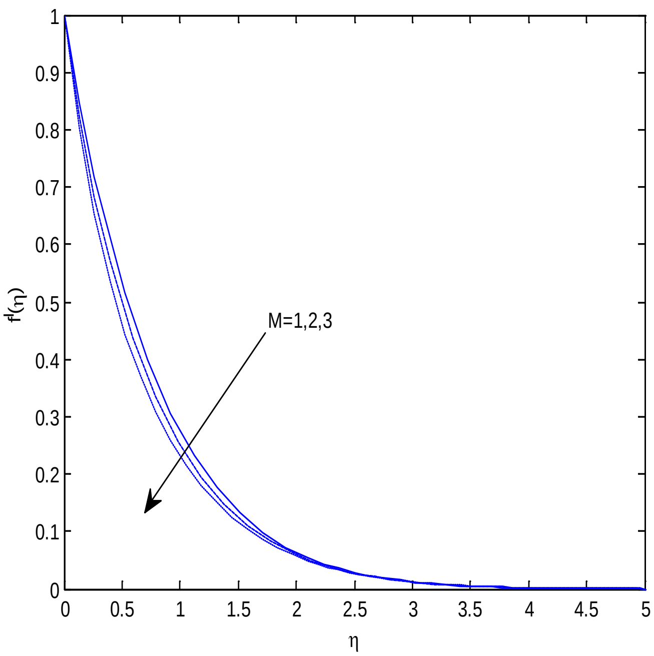

Fig. 3 Velocity profiles for different values of magneticfield parameter M

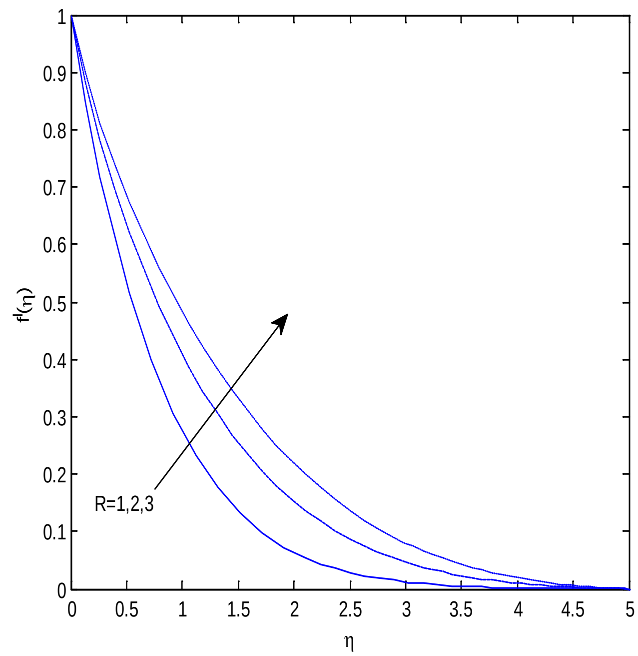

Fig. 5 Velocity profiles for different values of radiation parameter R

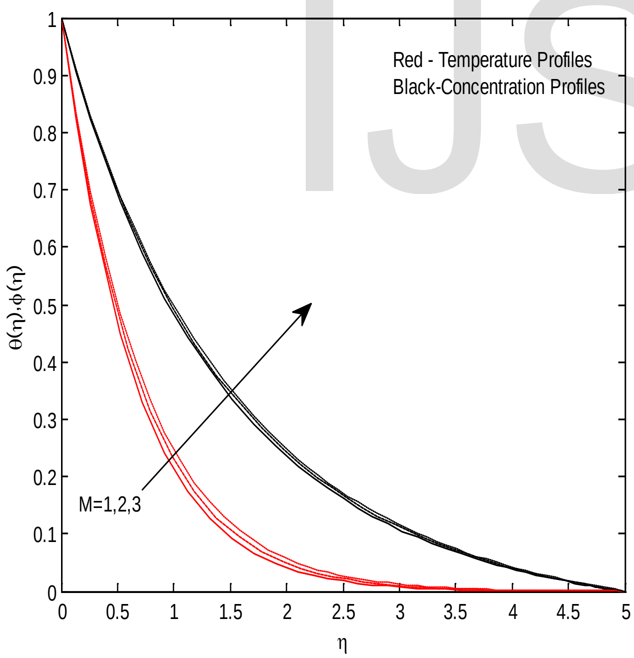

Fig. 4 Temperature and Concentration profiles for different values of magneticfield parameter M

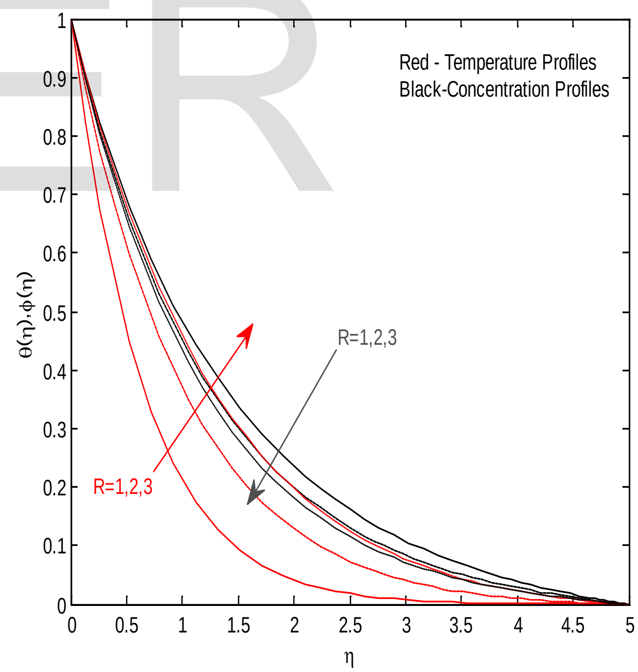

Fig. 6 Temperature and concentration profiles for different values of radiation parameter R

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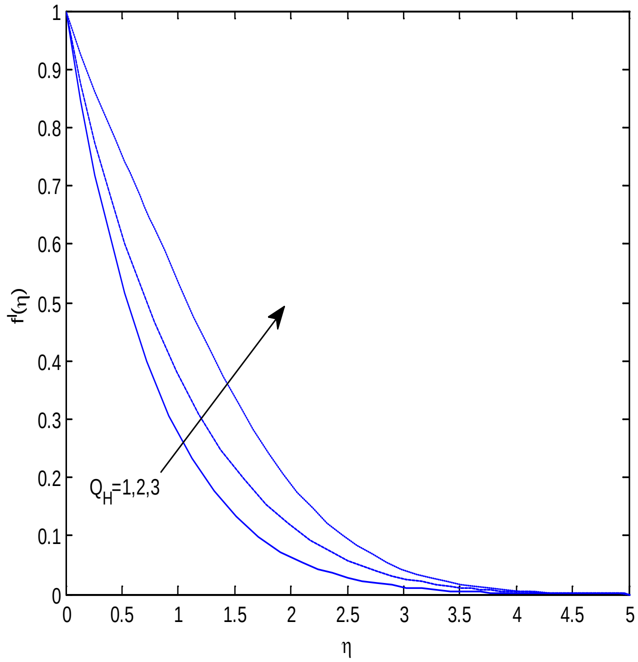

Fig. 7 Velocity profiles for different values of heat source parameter Q_H

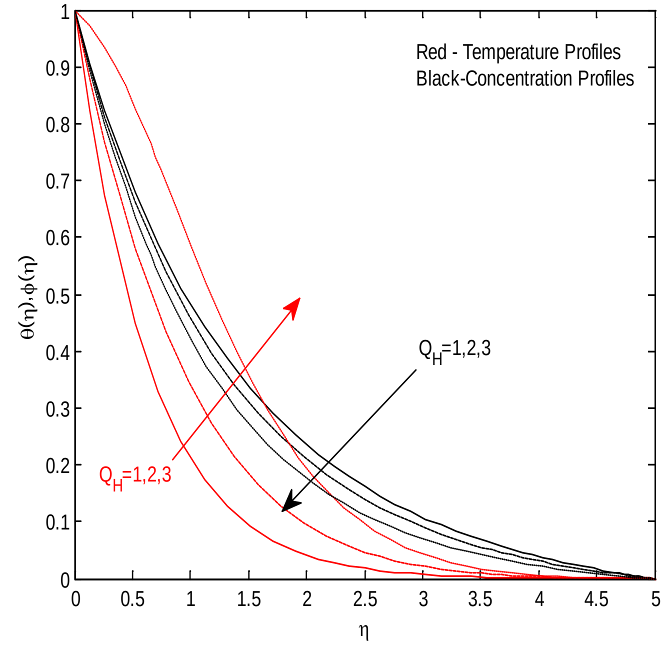

Fig. 8 Temperature and concentration profiles for different values of heat source parameter Q_H

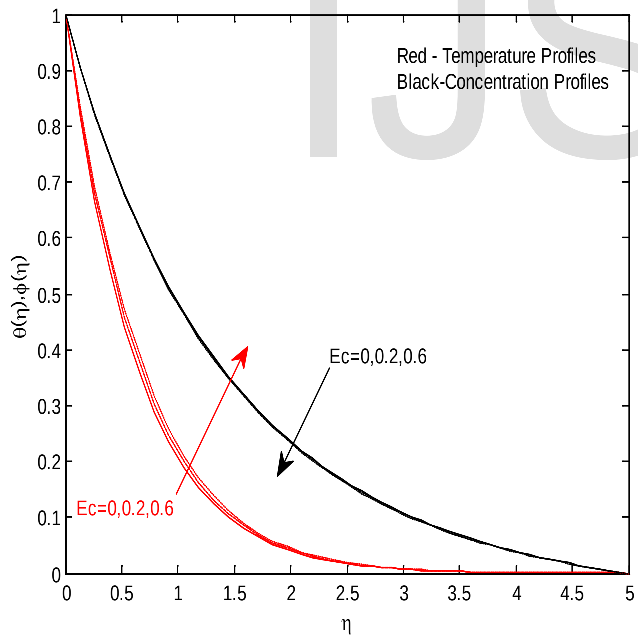

Fig. 9 Temperature and concentration profiles for different values of viscous dissipation parameter Ec

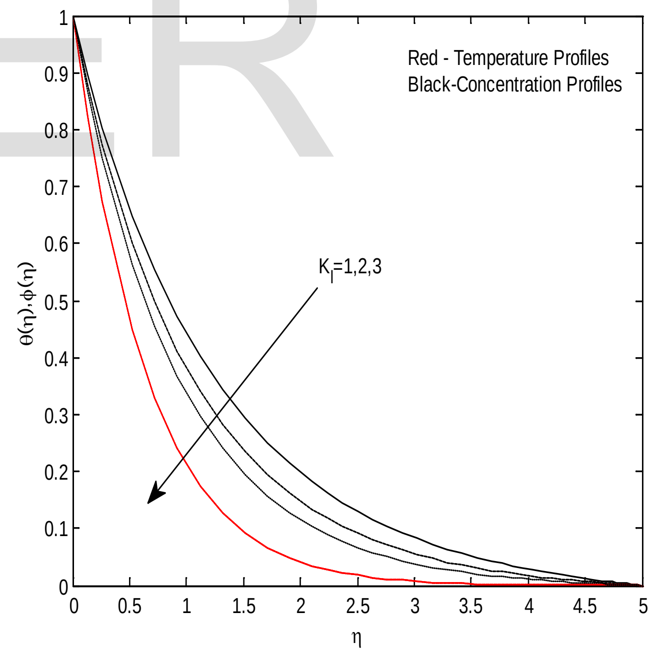

Fig. 10 Temperature and concentration profiles for different values of Chemical reaction parameter K_l

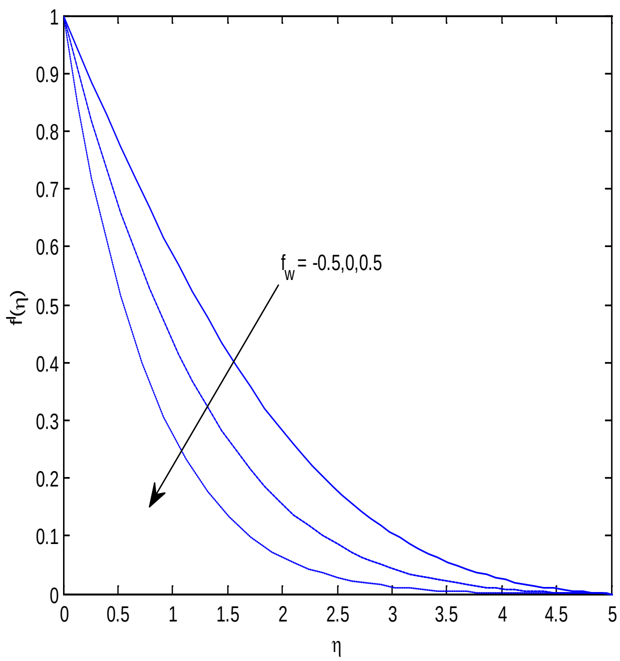

Fig. 11 Velocity profiles for different values of suction/injection parameter f_w

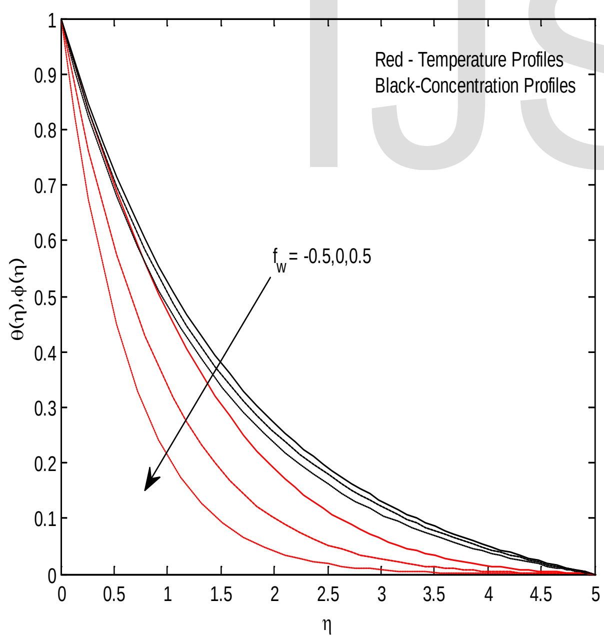

Fig. 12 Temperature and concentration profiles for different values of suction/injection parameter f_w

Conclusions

This paper presents heat and mass transfer in MHD nanofluid flow over an exponentially stretching surface in presence of internal heat generation/absorption, chemical reaction, radiation and suction/injection effects. The governing partial differential equations of the flow are converted into nonlinear coupled ordinary differential equations by using similarity transformation and then solved numerically. The effect of non-dimensional parameters on velocity, temperature and concentration profiles are discussed and presented through graphs. The findings of the numerical results are summarized as follows:

- Exponential parameter depreciates the velocity profiles, temperature and concentration profiles.
- Increase in magneticfield parameter causes to depreciate in velocity profiles of the flow and improves the thermal boundary layer.
- Radiation, heat generation/absorption parameters have capability to enhance the momentum and thermal boundary layers.
- Suction/injection parameter declines the velocity, temperature and concentration boundary layer thickness.

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References

[1] A.Chakrabarti and A.S.Gupta, "Hydromagnetic flow and heat transfer over a stretching sheet", *Quarterly of Applied Mathematics*, 37, 73-98, 1979.
[2] I.Ahmad, M.Sajid, W.Awan, M.Rafique, W.Aziz, M.Ahmed, A.Abbasi and M.Taj, "MHD flow of a viscous fluid over an exponentially stretching sheet in a porous medium", *J. of .App. Mathematics*, Vol 2014, 1-7, 2014.
[3] A.M.Rohni, S.Ahmad, I.Pop, "Flow and heat transfer at a stagnation point over an exponentially shrinking vertical sheet with suction ", *Int.J.Thermal Sciences*, 75, 164-170, 2014.
[4] N.Sandeep, A.V.B Reddy, V. Sugunamma, "Effect of radiation and chemical reaction on transient MHD free convective flow over a vertical plate through porous media", *Chemical and process engineering research*, 2, 1-9, 2012.

[5] N.Sandeep and V.Sugunamma, "Effect of inclined magneticfield on unsteady free convection flow of a dusty viscous fluid between two infinite flat plates filled by porous medium", *Int.J.App.Math.Modeling*, 1, 16-33, 2013.

[6] O.D. Makinde, A.Aziz, "Boundary layer flow of a nanofluid past a stretching sheet with convective boundary condition", Int.J.Ther.Sci, 50, 1326-1332, 2011.

[7] P.Mohankrishna, V.Sugunamma, N.Sandeep, "Radiation and magneticfield effects on unsteady natural convection flow of a nanofluid past an infinite vertical plate with heat source", *Chemical and Process Engineering Research*, 25, 39-52, 2014.

[8] N.Sandeep, V.Sugunamma, P. Mohankrishna, "Effects of radiation on an unsteady natural convective flow of a EG-Nimonic 80a nanofluid past an infinite vertical plate", *Advances in Physics Theories and Applications*, 23, 36-43, 2013.

[9] J.V.Ramana Reddy, V.Sugunamma, P.Mohan Krishna, N.Sandeep, "Aligned magneticfield, Radiation and chemical reaction effects on unsteady dusty viscous flow with heat generation/sbsorption", *Chemical and process eng. Research*, 27, 37-53, 2014.

[10] P.Mohan Krishna,V.Sugunamma and N.Sandeep, "Magneticfield and chemical reaction effects on convective flow of dusty viscous fluid", *Communications in App. Sciences*, 1, 161-187, 2013.

[11] F.M Hady, F.S Ibrahim, S.M Abdel-Gaied and M.R Eid, "Radiation effect on viscous flow of a nanofluid and heat transfer over a nonlinerly stretching sheet", *Nanoscale Research Letters*, 7, 299-310, 2014.

[12] N. Sandeep, V.Sugunamma, P.Mohan Krishna, "Aligned Magnetic Field, Radiation, and Rotation Effects on Unsteady Hydro magnetic Free Convection Flow Past an Impulsively Moving Vertical Plate in a Porous Medium", *Int. Journal of Eng. Mathematics*, http://dx.doi.org/10.1155/2014/565162, 2014.

[13] P.Rana, R.Bhargava, "Flow and heat transfer of a Nanofluid over a nonlinearly stretching sheet: a numerical study". *Commun Nonlinear Sci Nume Simulation*, 17, 212-226, 2014.

[14] K.Bhattacharya, "Boundary layer flow and heat transfer over exponentially shrinking sheet", *Chinese Phys Lett*, 28(7), 074701, 2011.

[15] M.Ferdows S.M.Chapal and A.A.Afify, "Boundary Layer Flow and Heat Transfer of a Nanofluid over a Permeable Unsteady Stretching Sheet with Viscous Dissipation", *Journal of Engineering Thermo physics* 23(3), 216–228, 2014.

[16] N.Sandeep and C.Sulochana, "Dual solutions of radiative MHD nanofluid flow over an exponentially stretching sheet with heat generation/absorption," App.Nano science. DOI 10.1007/s13204-015-0420-z, 2015.

[17] F.K.Murshed, R. Indranil, C. Arnab, N.Sandeep, Radiation Effect on Boundary Layer Flow of a Nanofluid over a Nonlinearly Permeable Stretching Sheet," Adv. Phy. Theories and appli., 40, 43-54, 2015.

[18] E.Jones, M.P.Brijgopal, K.R.Ravindra, N.Sandeep, "Aligned Magneticfield, Radiation and Chemical Reaction Effects on MHD Boundary Layer Flow over a Moving Vertical Porous Plate," Chemical and Process Engineering Research, 31, 89-103, 2015.

[19] C.S.K.Raju, N.Sandeep, M.J.Babu,V.Sugunamma, "Radiation and Chemical Reaction Effects on Thermophoretic MHD Flow over an Aligned Isothermal Permeable Surface with Heat Source", Chemical and Process Engineering Research, 31, 27-42, 2015.

[20] M. Subhas Abel, P.G.Siddheshwar, N.Mahesha, Numerical solution of the momentum and heat transfer equations for a hydromantic flow due to a stretching sheet of a non-uniform property micro polar liquid, Applied Mathematics and Computation, 217 (12) (2012) 5895-5909.